## SHEET NO (3)

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**Q1)**

Consider the code { 0, 10, 01 } for a ternary (D=3) source. Justify your answers to the following questions:

1. Is the code instantaneous?
2. Is the code nonsingular?
3. Is the code uniquely decodable?
4. Is there an instantaneous code with the same codeword lengths? If so, find an example

**Q2)**

Consider the code {0, 01}.

(a) Is it instantaneous?

(b) Is it uniquely decodable?

(c) Is it nonsingular?

**Q3)**

Prove that for a discrete memoryless source with entropy H(X) there is a D-ary free code with entropy HD(X) and the code will achieve $HD(X) \leq L < HD(X) + 1$ where E[L] is the expected length of the code.

**Q4)**

Determine which of the following codes is uniquely decodable:

(i) {0, 10, 11}

(ii) {0, 01, 11}

(iii) {0, 01, 10}

(iv) {0, 01}

(v) {00, 01, 10, 11}

(vi) {110, 11, 10}

(vii) {110, 11, 100, 00, 10}

**Q5)**

Let the random variable $X$ have five possible outcomes $\{1, 2, 3, 4, 5\}$. Consider two distributions $p(x)$ and $q(x)$ on this random variable.

| Symbol | $p(x)$ | $q(x)$ | $C_1(x)$ | $C_2(x)$ |
|---|---|---|---|---|
| 1 | $\frac{1}{2}$ | $\frac{1}{2}$ | 0 | 0 |
| 2 | $\frac{1}{4}$ | $\frac{1}{8}$ | 10 | 100 |
| 3 | $\frac{1}{8}$ | $\frac{1}{8}$ | 110 | 101 |
| 4 | $\frac{1}{16}$ | $\frac{1}{8}$ | 1110 | 110 |
| 5 | $\frac{1}{16}$ | $\frac{1}{8}$ | 1111 | 111 |

**(a)** Calculate $H(p)$, $H(q)$, $D(p||q)$, and $D(q||p)$.

**(b)** The last two columns represent codes for the random variable. Verify that the average length of $C_1$ under $p$ is equal to the entropy $H(p)$. Thus, $C_1$ is optimal for $p$. Verify that $C_2$ is optimal for $q$.

**(c)** Now assume that we use code $C_2$ when the distribution is $p$. What is the average length of the codewords. By how much does it exceed the entropy $p$?

**(d)** What is the loss if we use code $C_1$ when the distribution is $q$?